MATHEMATICAL DESCRIPTION OF THE SADOSA PROGRAM SYSTEM

by

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CONTENTS

O. INTRODUCTION

The SADOSA program system is designed for high precision computation of three-dimensional station coordinates using doppler observations of the satellites of Navy Navigation Satellite System. The program system consists of three programs: the SPPENC, and MERGE programs for preprocessing, and the SADOSA program for high precision geodetic adjustment.

The SPPENC program operates in single point mode. In a pass-by-pass manner the observations are edited by elevation angles and symmetry limits, a numerical filtering is performed by residual limits and single station coordinates are computed as well as data are edited and output for further processing. The residual editing is based on separate pass-by-pass least squares solution. The single station coordinates are adjusted by sequential least squares method. The satellite coordinates are obtained from broadcast ephemerides.

At the second stage the MERGE program arranges the SPPENC-edited data into one file in an orbit-by-orbit manner for final processing by SADOSA. For every orbit unique orbital data are computed. In case of broadcast ephemerides the unique orbit will be based on the union of broadcasted data coming from different stations. If precise ephemerides are available they replace the respective broadcast ephemerides. Finally, each orbit will be represented by coefficients of Chebyshev polinomials.

At the final stage a high precision geodetic adjustment of a multistation network is carried out by the SADOSA program using the merged file. Most of the principal properties of the SADOSA program system is realized at this stage, namely: multilocation short arc or semi-short arc method, range difference or pseudo-range model, different ways of weighting the observables and selection of coordinate system as well as constraining the station coordinates, the orbits and the error model parameters.

This document is a MATHEMATICAL DESCRIPTION which gives the mathematical models used in the programs of the SADOSA program system. The realization can be found in the PROGRAMMING DOCUMENTATION and in the program listings. How to use it in practice, the user is referred to the OPERATOR'S MANUAL.

The 5th version of the SADOSA program system is discussed in this document.

1. STATEMENT OF THE PROBLEM

A satellite transmits on frequency f_s. A ground station receiver will receive the Doppler shifted frequency f_r as follows:

$$f_r = f_s\left(1+\frac{1}{c}\frac{dr}{dt}\right)^{-1} \qquad /1/$$

where c is the light velocity in vacuum, dr/dt is the slant range velocity of transmitter relative to the receiver.

Let us denote the antenna phase centre coordinates by X_g, Y_g, Z_g /g=ground/, the coordinate of the satellite position at epoch t_i by X_{si}, Y_{si}, Z_{si} /s=satellite/ and the respective station-to-satellite slant range by Δr_i. Further, let Δt_i be the propagation time of a signal transmitted at epoch t_i, and f_g the receiver frequency. The integrated Doppler count N_i is obtained by counting the beat frequency $/f_g-f_r/$ between two epochs as expressed below:

$$N_i = \int_{t_{i-1}+\Delta t_{i-1}}^{t_i+\Delta t_i} (f_g-f_r)dt \qquad /2/$$

Using Eq. 1 and 2, a function F_1 can be found to express the range difference $\Delta r_i = r_i-r_{i-1}$ as function of measured doppler counts, time epochs and other data. Implicitely,

$$\Delta r_i = F_1\left(N_i, f_s, f_g, t_{i-1}, t_i, \Delta t_{i-1}, \Delta t_i, \ldots\right) \qquad /3/$$

From the other hand, the same range difference can be expressed as a function F_2 of the antenna and satellite coordinates as follows implicitely:

$$\Delta r_i = F_2\left(X_g, Y_g, Z_g, X_{si}, Y_{si}, Z_{si}, X_{si-1}, Y_{si-1}, Z_{si-1}\right) \qquad /4/$$

By means of the measured quantities and suitable approximate values of non-measured parameters and, further, linearizing the function $F_3=F_2-F_1$, thus obtaining observational equations, one can adjust for corrections to approximate parameters by least squares method.

2. ERROR MODEL

For practical use the explicite form of function F_1 is to be found. In connection with terms of Eq. 3 it has to be noticed that

- the frequencies f_s and f_g are continously varying by time,
- the above time epochs are generated by the varying f_s and f_g,
- the cycle count is performed not at the phase centre of the antenna but in other block of the receiver,
- the doppler counts are disturbed by atmospherical refractions, relativistic effects and Earth rotation effect.

These effects partly can be considered as corrections to be introduced and partly have to be determined as unknown error model parameters.

2.1. Frequency bias

The time varying receiver frequency is expressed by

$$f_g = f_{go} + \delta f_g(t_k) + \dot{f}_g(t_k)(t-t_k) \qquad /5/$$

where f_{go}, $\delta f_g(t_k)$ and $\dot{f}_g(t_k)$ are the nominal value, offset and drift of the receiver frequency at some initial epoch t_k. Similar formula can be written for the transmitter frequency:

$$f_s = f_{so} + \delta f_s(t_k) + \dot{f}_s(t_k)(t-t_k) \qquad /6/$$

In practice, at the starting level of processing /SPPENC program/ it is supposed that f_{go}=400 MHz and the δf_g is the offset from 400 MHZ to be determined. At higher level of processing /SADOSA program/ a good approximation is used for f_{go} which contains already an approximate correction $\overline{\delta f}_g$ and only a small correction $d\delta f_g$ has to be determined so that

$$\delta f_g = \overline{\delta f}_g + d\delta f_g \qquad /7/$$

In case of the transmitter frequency always a good nominal value f_{so} is available by the satellite message. However, in computation the δf_g and δf_s can not be separated, their composition is handled as a bias of the error model.

On a higher level of processing, the frequency drifts $\dot{f}_g$ and $\dot{f}_s$ are handled together, too, as a second bias of the error model.

2.2. Time corrections and bias

Let us consider Figure 1. The transmitter signal starts at epoch t_{1i} /first event/ and after propagation in the atmosphere comes to antenna phase centre of the receiver at epoch t_{2i} /second event/. Here the doppler and timing signal decoding begins. Decoding of the doppler signal requires time interval τ_{1i} and is finished by epoch t_{3i} /third event/. Decoding of the timing signal takes some time τ_{2i} and is finished by epoch t_{4i} /fourth event/. The cycle counting starts when the first positive zero crossing occurs at epoch t_{5i} /fifth event/. The interval between the fourth and fifth event is τ_{3i}. The interval τ_{2i} varies between 200-800 μs, τ_{1i} is much less than the previous. They are not measured, only a nominal value τ_g, called receiver delay, is available. The correction τ_{3i} is measured in case of JMR-type receivers, or negligible in case of other receivers.

Thus, the theoretical gating time epochs in the integral by Eq. 2 are t_{5i-1} and t_{5i} which can be connected with the corresponding signal emission time by the following equation:

$$t_{5i} = t_{1i} + \frac{r(t_{1i})}{c} + \tau_i \qquad /8/$$

where $r(t_{1i})$ is the phase centre-to-satellite slant range at epoch of signal emission t_{1i} and τ_i is the instantaneous time delay. Its value can be expressed by

$$\tau_i = \tau_{2i} + \tau_{3i} - \tau_{1i} \qquad /9/$$

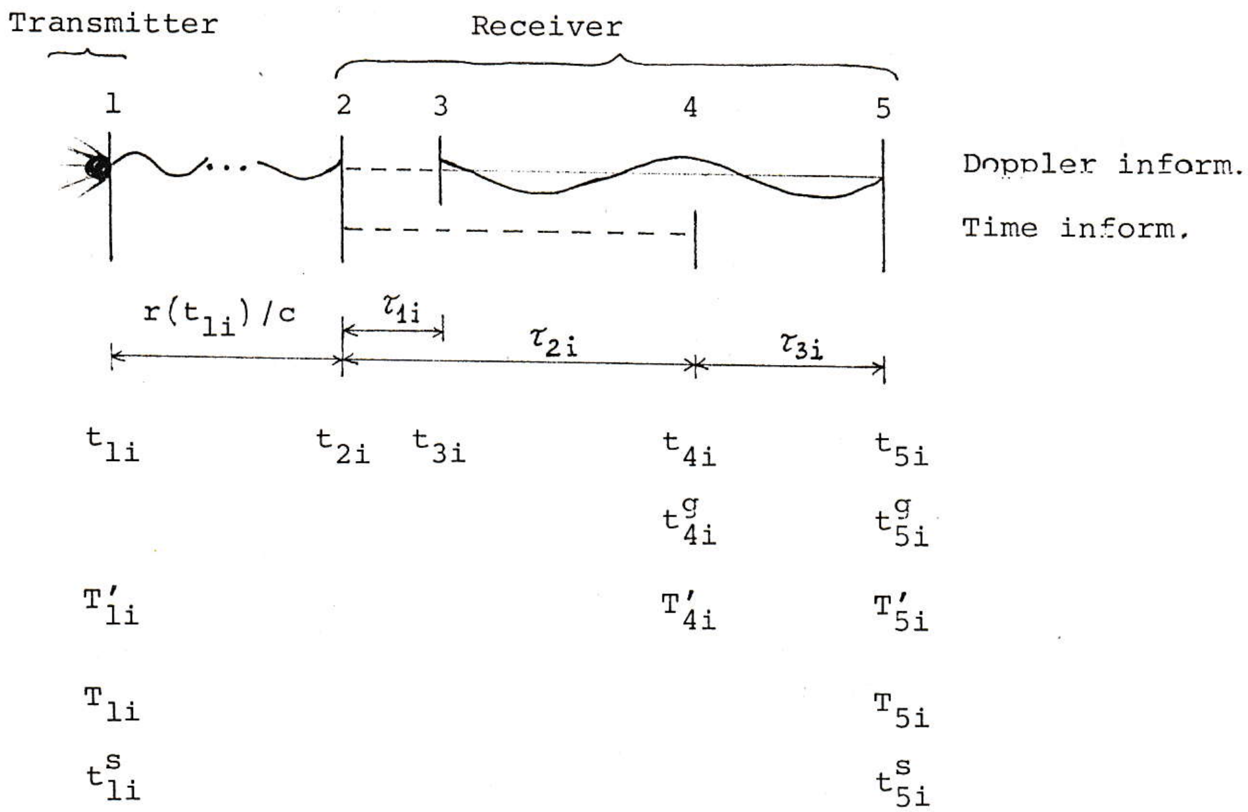


Figure 1.

In practice, a geodetic doppler receiver measures the epoch t^g_{5i} /and t^g_{4i}/ in the local time system t^g generated by the receiver oscillator as denoted on Figure 1. This system is in error by an offset time Δt^g_k and has a scale bias due to the offset frequency δf_g and the frequency drift $\dot{f}_g$ as expressed by the following formula to convert the local time system $t^{g'}$ to the uniform time system t:

$$t = t^g - \Delta t_k^g - \frac{\delta f_g(t_k)}{f_{go}}(t-t_k) - \frac{\dot{f}_g(t_k)}{2f_{go}}(t-t_k)^2 \qquad /10/$$

At the starting level of processing by SPPENC program the local time system t^g is transferred to the universal time represented by the first two-minute satellite time signal. The epoch of each event is obtained in system denoted by T′ on Figure 1. The nominal receiver delay τ_g and the time correction τ_{3i} are applied as corrections. The receiver synchronizes its clock to the first two-minute satellite time signal as to the signal with zero index "o" and the t_{4o}^g readout is obtained a epoch t_{4o}. The difference between the two systems t^g and t is

$$\Delta t_o^g = \frac{r(t_{1o})}{c} + \tau_{2o} - \tau_{1o} \qquad /11/$$

Neglecting the term $\Delta\tau$ in right side of the equation

$$\tau_{2o} - \tau_{1o} = \tau_g + \Delta\tau \qquad /12/$$

where $\Delta\tau$ is a correction to the nominal receiver delay, we obtain an approximant Δt_o to the correction Δt_o^g as follows

$$\Delta t_o = \frac{r(t_{1o})}{c} + \tau_g \qquad /13/$$

Then every doppler incrementation time measurement is computed in local time frame transferred to satellite time using the following formula:

$$T'_{5i} = t_{5i}^g + \Delta t_o \qquad /14/$$

The epoch of a fictive satellite signal corresponding to doppler incrementation can be computed by the following way:

$$T'_{1i} = T'_{5i} - \frac{r(t_{1i})}{c} - \tau_{3i} - \tau_g \qquad /15/$$

In Eq. 15 the $\Delta\tau$ is neglected and a good approximation of slant range r is used; further

$$\tau_{3i} = t^g_{5i} - t^g_{4i} \qquad /16/$$

Summarizing the starting level, the T'_{5i} and T'_{1i} epochs are used, the τ_g and τ_{3i} are considered as corrections.

For higher level of processing by SADOSA program the local time frame transferred to satellite frame, denoted as T', is scaled up using an approximate offset frequency $\overline{\delta f}_g$ and a quasi universal time system T is obtained by means of the following formula:

$$T_{5i} = T'_{5o} + (T'_{5i} - T'_{5o}) \frac{4 \cdot 10^8}{4 \cdot 10^8 + \overline{\delta f}_g} \qquad /17/$$

and

$$T_{1i} = T'_{1o} + (T'_{1i} - T^i_{1o}) \frac{4 \cdot 10^8}{4 \cdot 10^8 + \overline{\delta f}_g} \qquad /17a/$$

Summarizing the higher level processing, the T_{5i} and T_{1i} epochs are used, the τ_g and τ_{3i}, are considered as corrections and, further, a dt synchronization bias is adjusted which contains mainly the $\Delta\tau$ correction to receiver delay.

It is worth to mention that the above formulas are given for JMR-type receivers.

For receivers of other type it is considered that $\tau_{3i}=0$, that is $t_{4i}=t_{5i}$.

The only exception is the CMA-722B which does not perform any time measurement. In this case the satellite time frame t^s is used and the starting point is the satellite--bit-rate-derived epoch of the first event, denoted as t^s_{1i}. A fictive doppler incrementation epoch t^s_{5i} is computed by the following formula:

$$t^{s}_{5i} = t^{s}_{1i} + \frac{r(t_{1i})}{c} + \tau_g \qquad /18/$$

After that, the t^{s}_{1i} and t^{s}_{5i} are considered equivalent to T_{1i} and T_{5i}. Here $\tau_{3i}=0$, too.

2.3. Ionospherical refraction

A doppler count observed on one channel contains a rather big effect of the ionospherical refraction. Every receiver of geodetic type performs two-channel measurements by means of which the first order ionospherical effect can be eliminated.

The JMR receivers record the already corrected doppler counts. In case of receivers of other type two-channel doppler counts are recorded from which a ΔN_{ion} correction can be computed by standard formulas described in different literatures. Here, the reader is referred to /Ashkenazi 1977/ or /Brown, Trotter 1969/ or /Wells 1974/.

In further evalution a doppler count corrected for the first order ionospherical effect and transformed to 400 MHz channel will be denoted: N^{ion}_{i}.

2.4. Tropospherical refraction

Numerous algorithms can be used to correct a doppler count for tropospherical refraction. In the SADOSA program system **the** model of Black's approximation has been adopted being in good agreement with rigorous models even at 5^{o} of elevation /Kouba 1979/.

The formula to compute correction to a slant range is as follows:

$$dr_i = k_D\left[I(E_i,h_D,l_C)-b(E_i)\right] + k_W\left[I(E_i,h_W,l_C)-b(E_i)\right] \qquad /19/$$

where

$$I(E_i,h,l_c) = \left\{ 1-\left[\frac{\cos E_i}{1+(1-l_c)h/R_g}\right]^2 \right\}^{-1/2} \qquad /20/$$

$$l_c = 0.833+\left[0.076+0.00015(T_D-273)\right] \exp(-0.3E_i) \qquad /21/$$

$$b(E_i) = 1.92/(E_i^2+0.6) \qquad /22/$$

further

$$\left.\begin{aligned} k_D &= 155.2\cdot 10^{-7} \frac{P}{T_D} h_D \\ k_W &= 155.2\cdot 10^{-7} \frac{4810e}{T_D^2} h_W \end{aligned}\right\} \qquad /23/$$

$$\left.\begin{aligned} h_D &= 40136 + 148.72(T_D-273.16) \\ h_W &= 11000 \end{aligned}\right\} \qquad /24/$$

R_g is the station radius, E_i is the satellite elevation angle in degrees, P is the atmospherical pressure in mbar, T_D is the dry temperature in ^{o}K and e is the partial water vapour pressure in mbar.

The partial pressure is computed from dry temperature and humidity F /percent/100/:

$$e = 6.11\ F\ 10^{\left[7.5(T_D-273.3)/(T_D-35.8)\right]} \qquad /25/$$

The alternative formula for e when the dry and wet temperatures /T_D and T_W/ are given is as follows

$$e = e_W-4.5\cdot 10^{-4}(1+1.68\ 10^{-3}T_W)(T_D-T_W)P \qquad /25a/$$

where

$$e_W = 1013.246\left(\frac{373.16}{T_W}\right)^{5.02803} \cdot \exp\left[-g(T_W)\right]$$

and

$$g\,T_W = 18.19728q+0.0187265\ \ 1-\exp\ -8.03945q\ \ +$$

$$+\ 3.1813\cdot 10^{-7}\ \exp\ 26.1205\ \ 1-\frac{T_W}{373.16}\ \ -1$$

$$q = \frac{373.16}{T_W} - 1$$

A $d\Delta r_i$ tropospherical correction has to be introduced into the Δr_i range difference, as follows

$$\Delta_{Trop,i} = dr_i - dr_{i-1} \qquad /26/$$

None of the refraction models can express the real condition of the troposphere. This affects the tropospherical correction to have a systematic error which is approximately proportional to the elevation. The range difference Δr_i remains in error:

$$d\Delta_{Trop,i} = dK\left\{\left[\sin(E_i^2+4)^{1/2}\right]^{-1} - \left[\sin(E_{i-1}^2+4)^{1/2}\right]^{-1}\right\} \qquad /27/$$

where dK is the refraction bias parameter of the error model.

Summarizing, in the error model a $(dr_i - dr_{i-1})$ correction has to be considered and a dK refraction bias has to be determined.

2.5. Relativistic correction

The local and satellite clocks are used. They move with relativistic velocity as compared to each other and both special and general relativity will effect. The instantenous value of these effects /Wells 1974/:

$$f'_s = f_s\left(1-\frac{V_s}{c}\right)^{1/2}\cdot\left(1-\frac{\mu}{c^2}\,\frac{R_s-R_g}{R_sR_g}\right) \qquad /28/$$

from which a Δ_{Rel} correction can be computed for a range difference corresponding to a doppler count accumulated during dT_i time interval:

$$\Delta_{Rel,i} = \left[\frac{V_s^2}{2c} + \frac{\mu}{c}\left(\frac{1}{R_g} - \frac{2}{R_{si}+R_{si-1}}\right)\right] dT_i \qquad /29/$$

where $V_s \approx 7500$ m/s is the satellite velocity, c is the velocity of light, $\mu=3.99\ 10^4\ m^3/s^2$ is the gravitational constant, R_g is the station distance from geocentre, R_{si} and R_{si-1} are the satellite distances from geocentre at epochs T_{1i} and T_{1i-1}, further $dT_i=T_{1i}-T_{1i-1}$.

It has been pointed out that the relativistic corrections are additive with the f_g-f_s frequency offset /Brown 1969 and Well 1974/. Thus, neglecting this correction won't influence the station coordinates rather than value of f_g.

2.6. Error model for range differences

Starting with Eq. 2 and using the basic equations 1,5,6,8,10, the following error model has been derived for a range difference with index i from station g to satellite pass s:

$$c_{1i}df+c_{2i}d\dot{f}+c_{3i}dt+c_{4i}dK=-\frac{c}{f_{so}}\left(N_i^{ion}-\Delta f dT_i\right)+\Delta_{\tau gi}+$$

$$+\Delta_{\tau 3i}+\Delta_{Trop,i}+\Delta_{Rel,i}+\Delta r_i \qquad /30/$$

where, in addition to the previous explanations,

$\Delta f=f_{go}-f_{so}$ is the frequency offset,

$dT_i = T_{5i}-T_{5i-1}$ is the integration interval of the doppler count, $\approx$30 sec,

$\Delta r_i=r(T_{1i})-r(T_{1i-1})$ is the range difference,

N_i^{ion} is the doppler count accumulated for dT_i and corrected for the first order ionospherical effect,

c is the velocity of light in vacuum,

$df = \frac{f_{so}}{f_{go}} (\delta f_g - \delta f_s)$ is the offset frequency unknown,

$d\dot{f} = \frac{f_{so}}{f_{go}} (\dot{f}_g - \dot{f}_s)$ is the frequency drift unknown,

dt is the synchronization bias, generally close to the correction $\Delta\tau$ of the receiver delay,

dK is the tropospherical factor in m.

The coefficients are as follows:

$$c_{1i} = - \frac{c}{f_{so}} dT_i$$

$$c_{2i} = - \frac{c}{2f_{so}} \left[(T_{5i} - T_{5k})^2 - (T_{5i-1} - T_{5k})^2 \right]$$

$$c_{3i} = - (\dot{r}_{1i} - \dot{r}_{1i-1})$$

$$c_{4i} = - \left\{ \left[\sin(E_i^2 + 4)^{1/2} \right]^{-1} - \left[\sin(E_{i-1}^2 + 4)^{1/2} \right]^{-1} \right\}$$

here $\dot{r}_{1i}$ and $\dot{r}_{1i-1}$ are the slant range rates at epochs T_{1i} and T_{1i-1}, respectively, and T_{5k} is the epoch corresponding to the middle of the orbit.

The corrections are as follows:

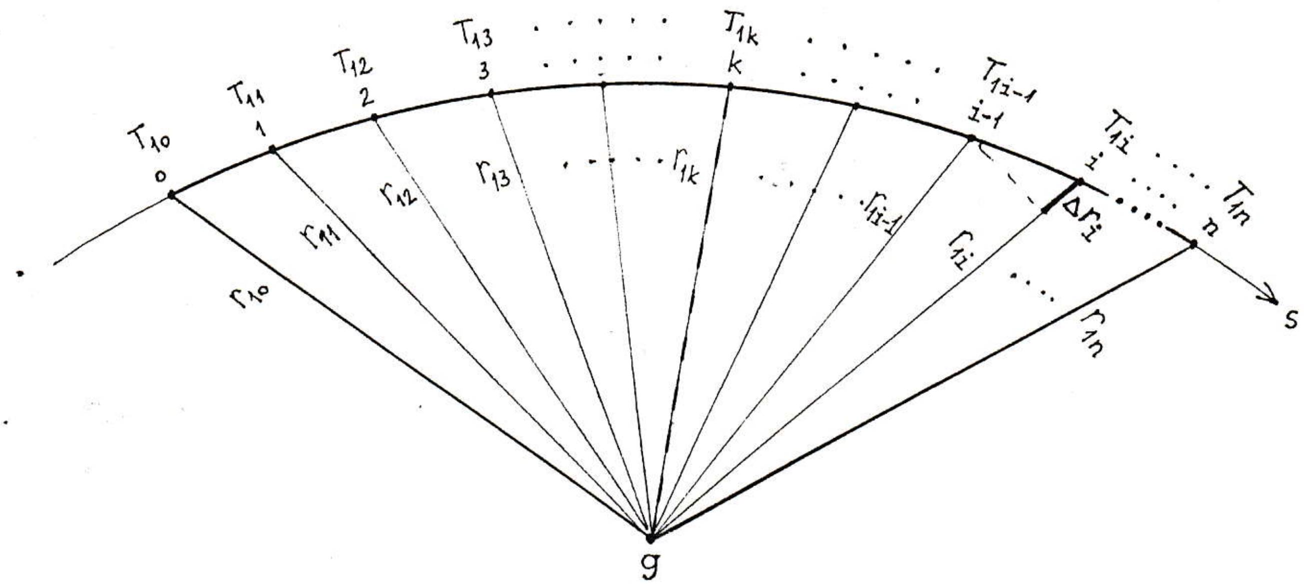
$\Delta_{\tau gi} = (\dot{r}_{1i} - \dot{r}_{1i-1}) \cdot \tau_g$, correction for the receiver delay,

$\Delta_{\tau 3i} = r_{1i} \cdot \tau_{3i} - r_{1i-1} \cdot \tau_{3i-1}$, special time correction which increases the accuracy of computations and enables the user to highly exploit one of the properties of JMR-type receivers,

$\Delta_{Trop,i} = dr_i - dr_{i-1}$, the tropospherical correction by Eq.19 and 26.

$\Delta_{Rel,i}$, the relativistic correction by Eq. 29.

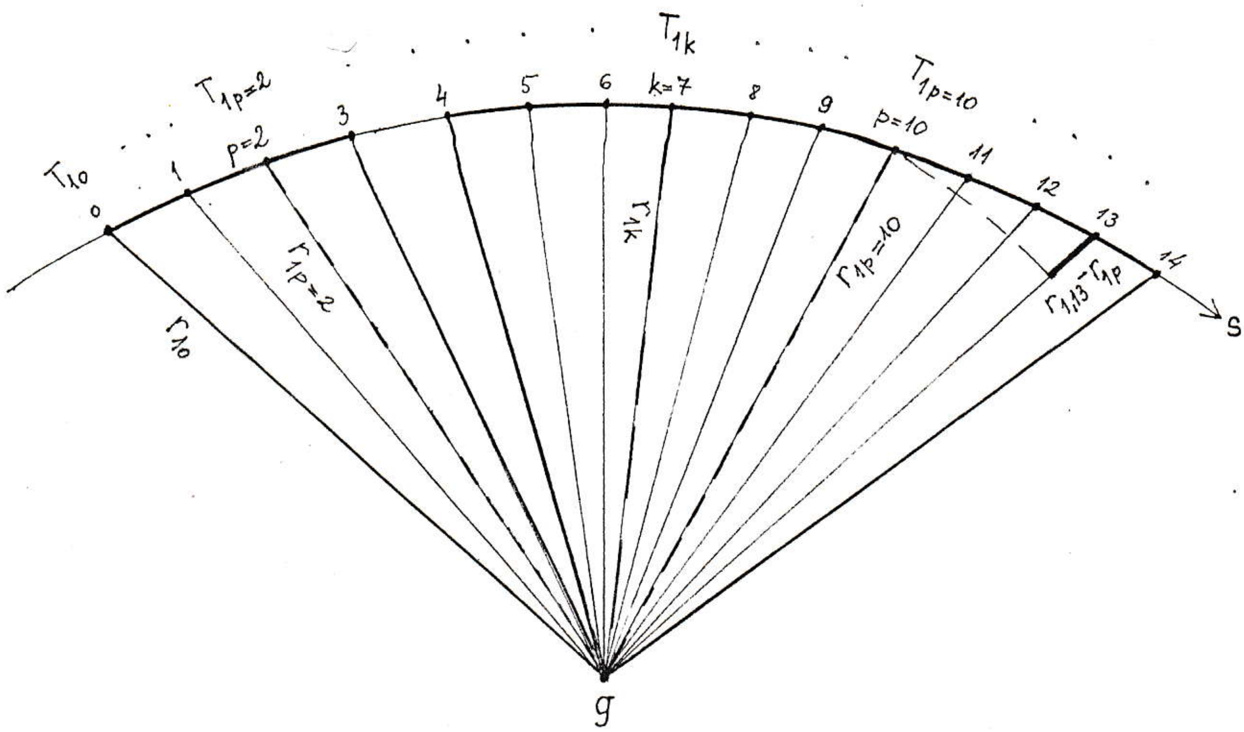
Figure 2 shows the configuration and epochs for range difference model.



$T_{5o}, T_{51}, T_{52}, T_{53}, \ldots, T_{5k}, \ldots T_{5i-1}, T_{5i}, \ldots, T_{5n}$

Figure 2.

Configuration and epochs for range difference model



$T_{1o}, \ldots, T_{1p=2}, \ldots, T_{1k}, \ldots T_{1p=1o}, \ldots$

Figure 3.

Configuration and epochs for pseudo-range model

2.7. Error model for pseudo-range method

For the observations to a pass which is interrupted not more than three times the pseudo-range model is indroduced as an alternative of solving the adjustment by SADOSA. In this case the $D_{i,p}$ doppler counts are used which have been continuously accumulated with respect to the middle point p /with epoch T_{5p}/ of a continuous part of the observed orbit. The number of such orbit parts is three as maximum.

The following error model is derived for this case:

$$c_{1ip}df+c_{2ip}d\dot{f}+c_{3i}dt+c_{4ip}dK = -\frac{c}{f_{so}}\left(D_{ip}^{ion}-\Delta f dT_{ip}\right)+$$

$$+\Delta_{\tau gip}+\Delta_{\tau 3ip}+\Delta_{Trop,ip}+\Delta_{Rel,ip}+r(T_{1i})-r(T_{1p}) \qquad /31/$$

In addition to the above explanations:

p is number of parts of an orbit, p=1,2,3, max(p) = 3

$dT_{ip}=T_{5i}-T_{5p}$

$$D_{ip}^{Ion} = \sum_{j=p}^{i} N_j^{Ion} \quad \text{/with appropriate sign/}$$

$$c_{1ip} = -\frac{c}{f_{so}}\cdot dT_{ip}$$

$$c_{2ip} = -\frac{c}{2f_{so}}\left[(T_{5i}-T_{5k})^2-(T_{5p}-T_{5k})^2\right]$$

$$c_{3ip} = -(\dot{r}_{1i}-\dot{r}_{1p})$$

$$c_{4ip} = -\left\{\left[\sin(E_i^2+4)^{1/2}\right]^{-1}-\left[\sin(E_p^2+4)^{1/2}\right]^{-1}\right\}$$

$$\Delta_{\tau gip} = (\dot{r}_{1i}-\dot{r}_{1p})\cdot\tau_g$$

$$\Delta_{\tau 3ip} = \dot{r}_{1i}\tau_{3i}-\dot{r}_{1p}\tau_{3p}$$

$$\Delta_{Trop,ip} = dr_i-dr_p$$

$\Delta_{Rel,ip}$ = Eq. 29 where the index i-1 must be replaced by p.

Figure 3. shows the configuration and epochs for pseudo--range model.

2.8. Remarks

The principal statements and formulas in Section 1. and Section 2 are commonly known. Where not mentioned, the reader is referenced to the basic sources which have been used to the error model: /Ashkenazi 1977/, /Brown, Trotter 1969/ and /Wells 1974/.

However, there are two points which are peculiar in the above error model derivation:

1. The time frame definition is based on
   - the really measured epoch of doppler incrementations $/T_{5i}/$,
   - the corresponding fictive signal emission $/T_{1i}/$ at the satellite computed from T_{5i},
   - transferring the local system to a quasi universal time system by means of the accurate two-minute satellite time signal.
2. A special time correction is used which increases the accuracy of computations and enables the user to highly exploit one of the properties of JMR-type receivers.

3. STATION COORDINATES

A slant range difference $\Delta r_i = r(T_{1i}) - r(T_{1i-1})$ can be expressed as follows:

$$\Delta r_i = \left[(X_{1i}-X_g)^2 + (Y_{1i}-Y_g)^2 + (Z_{1i}-Z_g)^2\right]^{1/2} -$$
$$- \left[(X_{1i-1}-X_g)^2 + (Y_{1i-1}-Y_g)^2 + (Z_{1i-1}-Z_g)^2\right]^{1/2} \qquad /32/$$

where X_g, Y_g, Z_g are rectangular coordinates of the station g, and X_1, Y_1, Z_1 with indices i and i-1 are the Earth fixed rectangular coordinates of the respective satellite points computed for epochs T_{1i} and T_{1i-1} given by Eq. 17a.

A set of corrections dX_g, dY_g, dZ_g are to be computed in the adjustment. Linearizing the Eq. 32 the following expression is obtained for later use in the observational equations:

$$\Delta r_i = \Delta r_i^c + a_{1i} dX_g + a_{2i} dY_g + a_{3i} dZ_g \qquad /33/$$

Here Δr_i^c is the slant range difference computed by means of preliminary station coordinates and Earth fixed satellite coordinates using Eq 32. The coefficient a_{1i}, a_{2i}, a_{3i} are the partial derivatives of Δr_i with respect to X_g, Y_g, Z_g expressed by the following equations of direction cosine differences:

$$a_{1i} = (X_{1i}^c - X_g^c)/r^c(T_{1i}) - (X_{1i-1}^c - X_g^c)/r^c(T_{1i-1})$$
$$a_{2i} = (Y_{1i}^c - Y_g^c)/r^c(T_{1i}) - (Y_{1i-1}^c - Y_g^c)/r^c(T_{1i-1}) \qquad /34/$$
$$a_{3i} = (Z_{1i}^c - Z_g^c)/r^c(T_{1i}) - (Z_{1i-1}^c - Z_g^c)/r_c(T_{1i-1})$$

where the higher index c means "calculated" or "preliminary".

For pseudo-range model similar expressions can be written:

$$r(T_{1i})-r(T_{1p})=\left[(X_{1i}-X_g)^2+(Y_{1i}-Y_g)^2+(Z_{1i}-Z_g)^2\right]^{1/2}$$

$$-\left[(X_{1p}-X_g)^2+(Y_{1p}-Y_g)^2+(Z_{1p}-Z_g)^2\right]^{1/2} \qquad /35/$$

$$r(T_{1i})-r(T_{1p})=r^c(T_{1i})-r^c(T_{1p})+a_{1ip}dX_g+a_{2ip}dY_g+a_{3ip}dZ_g \qquad /36/$$

$$a_{1ip}=(X^c_{1i}-X^c_g)/r^c(T_{1i})-(X^c_{1p}-X^c_g)/r^c(T_{1p})$$

$$a_{2ip}=(Y^c_{1i}-Y^c_g)/r^c(T_{1i})-(Y^c_{1p}-X^c_g)/r^c(T_{1p}) \qquad /37/$$

$$a_{3ip}=(Z^c_{1i}-Z^c_g)/r^c(T_{1i})-(Z^c_{1i}-Z^c_g)/r^c(T_{1p})$$

In case of corrections to latitude /dφ, rad/, longitude /dλ ,rad/ and ellipsoidal height /dH,m/ the coefficient in Eq. 34 have to be transformed by

$$G_g = \begin{bmatrix} -R_g\sin\varphi_g\cos\lambda_g & -R_g\cos\varphi_g\sin\lambda_g & \cos\varphi_g\cos\lambda_g \\ -R_g\sin\varphi_g\sin\lambda_g & R_g\cos\varphi_g\cos\lambda_g & \cos\varphi_g\sin\lambda_g \\ R_g\cos\varphi_g & 0 & \sin\varphi_g \end{bmatrix} \qquad /38/$$

so that

$$\left[\bar{a}_{1i},\bar{a}_{2i},\bar{a}_{3i}\right] = \left[a_{1i},a_{2i},a_{3i}\right]\cdot G_g \qquad /39/$$

4. ORBIT REPRESENTATION

Two sources of satellite coordinates are used in the SADOSA program system: broadcast ephemeris /BE/ and precise ephemeris /PE/.

4.1. Precise aphemeris

The precise ephemerides are computed in special cases for satellites and are available on special request. The precise ephemerides consist of the satellite rectangular coordinates X_i, Y_i, Z_i /in m/ and their respective velocity /in m/s/ computed for round minute epochs of UT /Ashkenazi 1977/. Their accuracy is estimated: 2m for each component of the rectangular coordinates and 0.001 m/s for each component of velocity.

For getting the satellite coordinates of the epochs required in the adjustment, the coordinates X_i, Y_i, Z_i are fit by least squares method to Chebyshev time polinomials, then 10 coefficients of fit are obtained and used.

The following Chebyshev polinomials are computed:

$$\begin{aligned} t_{1,i} &= 1 \\ t_{2,i} &= x_i \\ t_{3,i} &= 2x_i - 1 \\ &\vdots \\ t_{10,i} &= 2x_i t_{9,i} - t_{8,i} \end{aligned} \qquad /40/$$

where x_i is the time argument which spans 30 minutes and is normalized to ± 1:

$$x_i = \frac{2(i-1)}{30-1} - 1 , \qquad i=1,2,\dots,30 \qquad /41/$$

A design matrix T is obtained the size of which is 30*10:

$$\underset{30,10}{T} = \begin{bmatrix} t_{1,1} & t_{2,1} & \cdots & t_{10,1} \\ t_{1,2} & t_{2,2} & \cdots & t_{10,2} \\ \vdots & \vdots & & \vdots \\ t_{1,30} & t_{2,30} & \cdots & t_{10,30} \end{bmatrix} \qquad /42/$$

The normal equations are

$$\underset{10,10}{N} = \underset{10,30}{T^T} \cdot \underset{30,10}{T} \qquad /43/$$

After getting the inverse N^{-1} of the above normal equations, the Chebyshev coefficients are obtained for X,Y and Z coordinates:

$$\underset{10,1}{C_X} = \underset{10,10}{N^{-1}} (\underset{10,30}{T^T} \cdot \underset{30,1}{X}) \qquad /44/$$
$$C_Y = N^{-1} (T^T \cdot Y)$$
$$C_Z = N^{-1} (T^T \cdot Z)$$

where for example

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{30} \end{bmatrix} \qquad /45/$$

The standard deviations of fit by the three components are obtained as follows:

$$\sigma_X^2 = \frac{(TC_X-X)^T (TC_X-X)}{30-1}$$

$$\sigma_Y^2 = \frac{(TC_Y-Y)^T (TC_Y-Y)}{30-1} \qquad /46/$$

$$\sigma_Z^2 = \frac{(TC_Z-Z)^T (TC_Z-Z)}{30-1}$$

Precise ephemerides with $\sigma_i > 1$ m /i=X,Y,Z/ are deleted.

At the required epochs of signal transmission /Eq.17a/

$$\underset{n,1}{T_1} = \begin{bmatrix} T_{11} \\ T_{12} \\ \vdots \\ T_{1i} \\ \vdots \\ T_{1n} \end{bmatrix}. \qquad /47/$$

the satellite coordinates are obtained in the following way. The Chebyshev polinomials are computed for the vector T_1 using the Eq. 40 with time argument

$$x_i' = \frac{2(T_{1i}-1)}{30-1} - 1 \qquad /48/$$

and similar to Eq. 40 and Eq. 42 a design matrix T′ of size $n*10$ is obtained /n=number of observations/.
The coordinates in question are computed:

$$X_1 = T'C_X$$
$$Y_1 = T'C_Y \qquad /49/$$
$$Z_1 = T'C_Z$$

where, for example

$$X_1 = \begin{bmatrix} X_{11} \\ X_{12} \\ \vdots \\ X_{1i} \\ \vdots \\ X_{1n} \end{bmatrix} \qquad /50/$$

In the above computations the positional data are in meters, the time epochs are in minutes.

4.2. Broadcast ephemeris

The NNSS satellites transmit their predicted orbital parameters from which the satellite coordinates can be computed at a required epoch. The accuracy of broadcast ephemerides is estimated as 26 m, 10 m and 5 m along the orbit, in out-of plane component and in radial direction, respectively.

The broadcast ephemeris consists of two parts:
- fixed satellite message to compute mean orbit
- variable satellite message to compute time varying corrections to the mean orbit.

The SADOSA program system is accounting for majority voted and coded satellite message. Decoding is carried out in a standard way described for example in /Ashkenazi 1977/ or /Wells 1974/. After decoding, the orbital data are in suitable form.

The fixed orbital data are as follows:

tp - Time of perigee in min
n - mean motion in rad/min
ω - argument of perigee at tp in rad.
$\dot{\omega}$ - rate of change of ω in rad/min
e - excentricity
a_o - semi-major axis in m
Ω - right ascension of ascending node in rad
$\dot{\Omega}$ - rate of change of Ω in rad/min
cosi - cosine of angle of inclination
λ_G - right ascension of Greenwich in rad
ID - satellite identification number
t_{inj} - last injection time in min
sini - sine of inclination
Δf_s - satellite frequency offset in Hz

The decoded variable parameters are as follows:

ΔE_j - Excentric anomaly correction /along track correction/ in radians for epochs t_o-2, t, t_o+2, t_o+4, ... where t_o is the emission time of lock-on signal in minutes. Altogether $j=1,2,3,\ldots,n_v$ corrections are available.

Δa_j - Semi-major axis corrections in meters for epochs as above and $j=1,2,3,\ldots n_v$.

$\Delta\eta_j$ - Out of plane correction in meters for epochs t_o-2, t_o+2, t_o+6,... if t_o is not devisable with 4 or t_o, t_o+4, t_o+8,... if t_o is devisable with 4. Altogether $j=1,2,3,\ldots, m_v$ corrections are available.

From the variable parameters ΔE_j, and Δa_j given at round 2-minute epochs, corrections ΔE_i, and Δa_i are computed at the satellite signal transmission epochs T_{1i} using a least squares 6-coefficient approximation by the base function $\phi_1=[1, t, t^2, t^3, t^4, t^5]$
t is the variable parameter epoch with respect to the lock-on

signal epoch T_{50}/. The same procedure is used for the out of plane corrections $\Delta\eta_j$ /j=4-minute epochs/ and $\Delta\eta_i$. Here the base function is $\phi_2 = [1, t, \cos 2nt, \sin 2nt]$ /n is the mean motion/. As result of the fit, 6- and 4-coefficient vectors $C_{\Delta E}$, $C_{\Delta a}$, $C_{\Delta\eta}$ are obtained and the standard deviations of fit $\sigma_{\Delta E}$, $\sigma_{\Delta a}$, $\sigma_{\Delta\eta}$ are computed. The corrections at epoch T_{1i} base function, in implicite form:

$$\Delta E_i = f(C_{\Delta E}, \phi_1(T_{1i}, T_{50}, n))$$
$$\Delta a_i = f(C_{\Delta a}, \phi_1(T_{1i}, T_{50}, n)) \qquad /51/$$
$$\Delta\eta_i = f(C_{\Delta\eta}, \phi_2(T_{1i}, T_{50}, n))$$

Finally, the Earth fixed satellite coordinates at the satellite signal transmission epoch T_{1i} are calculated as functions of fixed parameters P_f, variable parameters and epochs T_{1i}, and T_{5o}:

$$X_{1i} = f_1(P_f, \Delta E_i, \Delta a_i, \Delta\eta_i, T_{1i}, T_{5o})$$
$$Y_{1i} = f_2(P_f, \Delta E_i, \Delta a_i, \Delta\eta_i, T_{1i}, T_{5o}) \qquad /52/$$
$$Z_{1i} = f_3(P_f, \Delta E_i, \Delta a_i, \Delta\eta_i, T_{1i}, T_{5o})$$

4.3. Precise v.v. broadcast ephemeris

At the starting process /that is in the SPPENC program/ only broadcast ephemerides are used and the satellite coordinates are computed by Eq. 51 and Eq. 52.

At higher processing level /the MERGE and SADOSA programs/ it is possible to use the precise and broadcast ephemerides alternatively. If precise ephemerides are available for an orbit then the satellite coordinates are computed as given in Section 4.2 by Eq. from 40 to 50. If no precise ephemerides are available or the precise ephemerides have been rejected due to standard deviations then another way is followed.

Each station has a separate set of the variable parameters for the given orbit. Generally, these sets differ from each other in that each set covers some different part of the orbit. The union taken from separate sets of the variable parameters will give the longest covering of the orbit by variable parameters. From the fixed orbital data and the union of variable parameters and using the Eq. 51 and 52, satellite X,Y,Z coordinates are computed at 1-minute epochs. The number of 1-minute epochs is

$$\ell = \mathrm{int}(\mathrm{int}(2*(T_{1k}-T_{5o}))/2)*2+2 \qquad /53/$$

where int means integer operation, T_{1k} and T_{5o} are epoch of the middle of observed orbit and epoch of the lock-on ; ℓ_{max} =30.

On basis of the 1-minute epochs and 1-minute satellite coordinates a Chebyshev fit is performed by Eq. 40-46. In this procedure instead of number of points 30 the number ℓ, obtained in Eq. 53, is everywhere substituted. Now the orbit is represented by coefficients C_X, C_Y and C_Z. In the SADOSA program the satellite coordinates are computed from these coefficients and using formulas Eq. 47-50 /here also 30 is substituted by ℓ/.

4.4. Short arc method

An orbit generated by either precise or broadcast ephemerides has a very accurate shape on a short arc of 15-20 minutes of time. However, the orbit itself is transferred and rotated with respect to basic coordinate system by such values which are comparable to the orbital standard deviations specified for the respective ephemeris type. The transfer and the rotation can be handled as bias parameters and determined as orbital unknowns /Wolf 1967; Brown, Trotter 1969/.

The coordinates of a satellite point at T_{1i} epoch can be expressed as follows:

$$X_{1i} = X_{1i}^{c} + dX_k + d\dot{X}_k(T_{1i}-T_{1k})$$
$$Y_{1i} = Y_{1i}^{c} + dY_k + d\dot{Y}_k(T_{1i}-T_{1k}) \qquad /54/$$
$$Z_{1i} = Z_{1i}^{c} + dZ_k + d\dot{Z}_k(T_{1i}-T_{1k})$$

where

$X_{1i}^c, Y_{1i}^c, Z_{1i}^c$ are preliminary coordinates computed from precise or broadcast ephemerides as described in Sections 4.1, 4.2 and 4.3,

dX_k, dY_k, dZ_k are positional corrections to orbit at epoch T_{1k} of middle of the orbit, they are to be computed in the least squares adjustment,

$d\dot{X}_k, d\dot{Y}_k, d\dot{Z}_k$ are velocity corrections at epoch T_{1k} of middle of the orbit to be computed in the least squares adjustment.

If only positional corrections are aimed to be adjusted then the adjustment method is called semi-short arc method. If both positional and velocity corrections are to be computed then the method is called short arc.

Contribution of the expressions by Eq. 54 to the range difference or the pseudo-range equations can be easy derived similar to the station coordinates unknows on the basis of Eq. 32 /Section 3./.

For range difference equations:

$$\Delta r_i = \Delta r_i^c + b_{1i}dX_k + b_{2i}dY_k + b_{3i}dZ_k + $$
$$+ b_{4i}d\dot{X}_k + b_{5i}d\dot{Y}_k + b_{6i}d\dot{Z}_k \qquad /55/$$

where $b_{1i} = -a_{1i}$

$b_{2i} = -a_{2i}$

$b_{3i} = -a_{3i}$

$$b_{4i} = -\left[dT_i(X_{1i}^c - X_g^c)/r^c(T_{1i}) - dT_{i-1}(X_{1i-1}^c - X_g^c)/r^c(T_{1i-1})\right]$$
$$b_{5i} = -\left[dT_i(Y_{1i}^c - Y_g^c)/r^c(T_{1i}) - dT_{i-1}(Y_{1i-1}^c - Y_g^c)/r^c(T_{1i-1})\right]$$

$$b_{6i} = -\left[dT_i\,(Z^c_{1i}-Z^c_g)/r^c(T_{1i}) - dT_{i-1}(Z^c_{1i-1}-Z^c_g)/r^c(T_{1i-1})\right]$$

$$dT_i = T_{5i} - T_{5k}$$

$$dT_{i-1} = T_{5i-1} - T_{5k}$$

For pseudo-range equations

$$r(T_{1i}) - r(T_{1p}) = r^c(T_{1i}) - r^c(T_{1p}) + b_{1ip}dX_k + b_{2ip}dY_k + b_{3ip}dZ_k + b_{4ip}d\dot{X}_k + b_{5ip}d\dot{Y}_k + b_{6ip}d\dot{Z}_k \qquad /56/$$

where

$$b_{1ip} = -a_{1ip}$$

$$b_{2ip} = -a_{\ ip}$$

$$b_{3ip} = -a_{\ ip}$$

$$b_{4ip} = -\left[dT_i\,(X^c_{1i}-X^c_g)/r^c(T_{1i}) - dT_p\,(X^c_{1p}-X^c_g)/r^c(T_{1p})\right]$$

$$b_{5ip} = -\left[dT_i\,(Y^c_{1i}-Y^c_g)/r^c(T_{1i}) - dT_p\,(Y^c_{1p}-Y^c_g)/r^c(T_{1p})\right]$$

$$b_{6ip} = -\left[dT_i\,(Z^c_{1i}-Z^c_g)/r^c(T_{1i}) - dT_p\,(Z^c_{1p}-Z^c_g)/r^c(T_{1p})\right]$$

$$dT_i = T_{5i} - T_{5k}$$

$$dT_p = T_{5p} - T_{5k}$$

When precise ephemerides used, then both positional and velocity correction are computed in rectangular coordinate system. In case of broadcast ephemeris along, out of plane and radial components are adjusted in meters. For this purpose the respective coefficient b_{1i}, b_{2i}, b_{3i} or b_{1ip}, b_{2ip}, b_{3ip} have to be transformed into the spherical system:

$$\left[\bar{b}_{1i}, \bar{b}_{2i}, \bar{b}_{3i}\right] = \left[b_{1i}, b_{2i}, b_{3i}\right] G_k$$

or

$$\left[\bar{b}_{1ip}, \bar{b}_{2ip}, \bar{b}_{3ip}\right] = \left[b_{1ip}, b_{2ip}, b_{3ip}\right] G_k \qquad /57/$$

where the transformation matrix

$$G_k = \begin{bmatrix} -\sin\varphi_k \cos\lambda_k & -\sin\lambda_k & \cos\varphi_k \cos\lambda_k \\ -\sin\varphi_k \sin\lambda_k & \cos\lambda_k & \cos\varphi_k \sin\lambda_k \\ \cos\varphi_k & 0 & \sin\varphi_k \end{bmatrix}$$

and R_k, φ_k, λ_k are the geocentrical distance, latitude and longitude of k-th satellite point /orbital mean/ at epoch T_{1k}.

5. OBSERVATIONAL EQUATIONS AND WEIGHT MATRICES

5.1. Observational equations

In case of a station i /i=g/ observing an orbit k, altogether n_{ik} observational equations are obtained one of which with index j /j=1,2,...,n_{ik}/ has, in general, the form below

$$a_{1j}dX_i+a_{2j}dY_i+a_{3j}dZ_i+b_{1j}dX_k+b_{2j}dY_k+b_{3j}dZ_k +$$
$$+b_{4j}d\dot{X}_k+b_{5j}d\dot{Y}_k+b_{6j}d\dot{Z}_k+c_{1j}df+c_{2j}d\dot{f}+c_{3j}dt+c_{4j}dK = l_j+v_j \qquad /58/$$

with quasi observable l_j equal to the right hand side of Eq. 30, and v_j residual.

This observational equation is composed of the effects of station unknowns /Eq. 33/, orbital unknowns /Eq. 55/ and error model parameters /Eq. 30/ and shows that the range difference model and the short arc method are used. Depending on the program and on the option wanted to be run, different compositions can be used in the program system:

- range difference model and short arc method /SADOSA/,
- range difference model and semi-short arc method /SADOSA/,
- pseudo-range model and short arc method /SADOSA/,
- pseudo-range model and semi-short method /SADOSA/,
- simplified range difference model and semi-short arc method /SPPENC/
- simplified range difference model with no orbital unknowns /SPPENC/

Let us introduce vectors of i-th station unknowns /$\hat{d}_i$/, k-th orbit unknowns /$\dot{d}_i$/ and error model unknowns of the given station-to-satellite event /$\ddot{d}_{ik}$/ so that.

$$\left.\begin{aligned}\hat{d}_i &= \left[dX_k,\ dY_k,\ dZ_k\right]^T\\ \dot{d}_k &= \left[dX_k,\ dY_k,\ dZ_k,\ d\dot{X}_k,\ d\dot{Y}_k,\ d\dot{Z}_k\right]^T\\ \ddot{d}_{ik} &= \left[df_{ik}, d\dot{f}_{ik}, dt_{ik}, dK_{ik}\right]^T\end{aligned}\right\} \qquad /59/$$

and the corresponding parts of design matrix:

$$\left.\begin{aligned}\hat{B}_{ik} &= \begin{bmatrix} \vdots & \vdots & \vdots \\ a_{1j} & a_{2j} & a_{3j} \\ \vdots & \vdots & \vdots \end{bmatrix}\\ \dot{B}_{ik} &= \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{1j} & b_{2j} & b_{3j} & b_{4j} & b_{5j} & b_{6j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}\\ \ddot{B}_{ik} &= \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ c_{1j} & c_{2j} & c_{3j} & c_{4j} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}\end{aligned}\right\} \qquad /60/$$

with $j = 1,2,\ldots,n_{ik}$,
and the vector of quasi observables and residuals

$$\begin{aligned}L_{ik} &= \left[l_1, l_2, \ldots, l_j, \ldots, l_{nik}\right]^T\\ V_{ik} &= \left[v_1, v_2, \ldots, v_j, \ldots, v_{nik}\right]^T\end{aligned} \qquad /61/$$

After that, the system of observational equations of the event of observations from station i to orbit k will have a matrix form:

$$\underset{n_{ik}*3}{\hat{B}_{ik}} \cdot \underset{3*1}{\hat{d}_i} + \underset{n_{ik}*6}{\dot{B}_{ik}} \cdot \underset{6*1}{\dot{d}_k} + \underset{b_{ik}*4}{\ddot{B}_{ik}} \cdot \underset{4*1}{\ddot{d}_{ik}} = \underset{n_{ik}*1}{L_{ik}} + \underset{n_{ik}*1}{V_{ik}} \qquad /62/$$

The other types of observational equations lead to matrix form similar to Eq. 62. Every further operation will carried out using Eq. 62.

5.2 Weight matrices

The quasi observable vector L_{ik} is associated with a respective weight matrix P_{ik} which can be computed optionally in different way.
For the adjustments in SPPENC program a unit matrix is used /single station, i index is omitted/:

$$\underset{n_k,n_k}{P_k} = E \qquad /63/$$

For range difference model, when a -0.5 correlation between quasi observables is supposed, the elements of P_{ik} are as follows /Graybill 1969/:

$$\left(P_{ik}\right)_{m,n} = \begin{cases} m\left(n_{ik}+1-n\right)\left[\left(n_{ik}+1\right) * \sigma^2/\sigma_o^2\right]^{-1}; & \text{if } n \geq m \\ n\left(n_{ik}+1-m\right)\left[\left(n_{ik}+1\right) * \sigma^2/\sigma_o^2\right]^{-1}; & \text{if } m \geq n \end{cases}$$

$m = 1,2,\ldots,n_{ik}$

$n = 1,2,\ldots,n_{ik}$

For pseudo-range equations, when no correlation is supposed, a diagonal matrix is introduced:

$$\underset{n_{ik}*n_{ik}}{P_{ik}} = \left(\sigma_o^2/\sigma^2\right)E \qquad /65/$$

In both range difference and pseudo-range model if stationary time series are supposed /Graybill 1969/ the weight matrix is as follows

$$\underset{n_{ik}*n_{ik}}{P_{ik}} = \frac{\sigma_o^2}{\sigma^2\left(1-\alpha^2\right)} \begin{bmatrix} 1 & -\alpha & 0 & \ldots & \ldots \\ -\alpha & 1+\alpha^2 & -\alpha & \ldots & \ldots \\ 0 & -\alpha & 1+\alpha^2 & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix} \qquad /66/$$

Where α is the first order coefficient of correlation of the observed quantities computed in the previous iteration. In the above equations σ_o is the a'priori standard deviation of unit weight selected to be equal to 1 or to 0.15, optionally. σ is the standard deviation of observations /in m/, which is either input separately for each station or determined in the previous iteration for each station-to-satellite event ik $(\sigma = \sigma_{ik})$.

5.3. Constraints

In each station-to-orbit event ik the error model parameters can be constrained on the basis of a'priori knowledge of their accuracy:

$$\underset{4*4}{W_{ik}} = \text{diag} \left\langle \sigma_{df}^{-2}, \sigma_{d\dot{f}}^{-2}, \sigma_{dt}^{-2}, \sigma_{dK}^{-2} \right\rangle \qquad /63/$$

For weight of the parameters which are not to be adjusted a zero value is supposed. If the time synchronization parameter dt is selected to be computed then for the time system definition it must be constrained, if $\sigma_{dt}=0$ is input then a default $\sigma_{dt} = 100\ \mu s$ is used.

The orbital unknowns also can be constrained. For a k-th orbit in case of broadcast ephemeris

$$\underset{6,6}{W_k} = \text{diag} \left\langle \sigma_{\Delta E}^{-2}, \sigma_{\Delta\eta}^{-2}, \sigma_{\Delta a}^{-2}, \sigma_{\dot{X}}^{-2}, \sigma_{\dot{Y}}^{-2}, \sigma_{\dot{Z}}^{-2} \right\rangle \qquad /64a/$$

$$\sigma_{\Delta E} = 26\ \text{m}, \quad \sigma_{\Delta\eta} = 10\ \text{m}, \quad \sigma_{\Delta a} = 5\ \text{m}$$

$$\sigma_{\dot{X}} = \sigma_{\dot{Y}} = \sigma_{\dot{Z}} = 0.01\ \text{m/s}$$

in case of precise ephemeris

$$\underset{6,6}{W_k} = \text{diag} \left\langle \sigma_{X}^{-2}, \sigma_{Y}^{-2}, \sigma_{Z}^{-2}, \sigma_{\dot{X}}^{-2}, \sigma_{\dot{Y}}^{-2}, \sigma_{\dot{Z}}^{-2} \right\rangle \qquad /64b/$$

$$\sigma_X = \sigma_Y = \sigma_Z = 2\text{m} \quad \text{and} \quad \sigma_{\dot{X}} = \sigma_{\dot{Y}} = \sigma_{\dot{Z}} = 0.001\ \text{m/s}$$

6. LEAST SQUARES SOLUTION

The observational equations are accumulated for every station-to-orbit event. The full matrix of observational equation all over the network would be obtained as follows:

$$AX + L = V \qquad \text{with } P \text{ weight matrix} \qquad /66/$$

The least squares minimum variance estimate of X is obtained by minimizing the Lagrange function

$$\varphi = V^T P V + X^T P_X X - 2K^T(AX+L-V) \qquad /67/$$

with respect to the parameters V, X and K. In the above equations for the total network

A is the design matrix
X,L and V are the vectors of unknowns, observables and residuals
P is the weight matrix
P_X is the a'priori weight matrix for the unknowns/constraints/.

After minimizing the φ and eliminating the unknowns K multiplier and V vectors, the estimation of X can be obtained from

values are built in which can be used optionally.
If the coordinate system definition is based on the satellite coordinates, then the weights must not be zero.
The station positions also can be constrained. If n_{st} is the number of all stations, the weight matrix will have the form:

$$\underset{n_{st} * n_{st}}{W} = \mathrm{diag}\left\langle \sigma_{X1}^{-2}, \sigma_{Y1}^{-2}, \sigma_{Z1}^{-2}, \ldots, \sigma_{X_{n_{st}}}^{-2}, \sigma_{Y_{n_{st}}}^{-2}, \sigma_{Z_{n_{st}}}^{-2} \right\rangle \qquad /65/$$

The σ-s may be of different values. If the system definition is based on station ccordinates, then the appropriate coordinates must be constrained. The σ-s generally are ∞ or 25 m.

The coordinate system in SADOSA can be defined alternatively by using the inner constraint approach /or free adjustment/ applied to the origin and the orientation of system.
Detailed description of the procedure is given, for example, in Blaha,1971 .

Further possibility is to constrain the relative position between m coordinate m= dX_{ij}, dY_{ij} and dZ_{ij} of selected stations i and j. If supposing that some m coordinate difference between stations i and j is known an accuracy of m_{ij} , then the respective element of the W_r r = relative matrix is as follows

$$w_{m_{ij}} = m_{ij}^{-2} \qquad /65a/$$

$$(A^TPA+P_X)X + A^TPL = 0 \quad /68/$$

$$X = -(A^TPA+P_X)^{-1}A^TPL \quad /69/$$

The variance-covariance matrix of the estimation is

$$\sum_X = \sigma_o^2(A^TPA+P_X)^{-1} \quad /70/$$

and the a'posteriori standard deviation of unit weight

$$\sigma_o^2 = \frac{V^TPV+X^TP_XX}{f} \quad /71/$$

where f is degree of freedom.

The above procedure is a principle way. It will be realized on three level of processing as shown below.

6.1. SINGLE STATION PASS-BY-PASS SOLUTION

In the SPPENC program the data filtering is based on a least squares solution for single station from single pass. Here the station coordinate and frequency offset corrections are unknowns. The range difference observational equation for j-th observation:

$$\bar{a}_{1j}d\varphi + \bar{a}_{2j}d\lambda + \bar{a}_{3j}dH + c_{1j}df = \bar{l}_j + \bar{v}_j \quad /72/$$

Here $\bar{a}_{1j}$, $\bar{a}_{2j}$, $\bar{a}_{3j}$ are computed as prescribed in Eq. 39 but using satellite positions computed for T'_{1j} epoch instead of T_{1j}; c_{1j} is computed as given in Eq. 30 but using T'_{5j} epoch instead of T_{5j}; $\bar{l}_j$ is computed as the right hand side of Eq. 30 but using in it T'_{1j} and T'_{5j} instead of T_{1j} and T_{5j} and the correction $\Delta_{Rel,j}$ is omitted.

For a single pass the observational equations for n measurements and the normal equations are computed:

$$\bar{A}X + \bar{L} = \bar{V}$$
$$\bar{A}^T\bar{A}X + \bar{A}^T\bar{L} = 0 \quad /73/$$

and the respective parameters are estimated:

$$\left[d\varphi\ ,\ d\lambda\ ,\ dH,\ df\right]^T = X = -(\bar{A}^T\bar{A})^{-1}\ \bar{A}^T\bar{L} \qquad /74/$$

Using the design matrix A and vector L, the estimated X and V vectors are computed, from which the standard deviation and the correlation coefficient for k-th pass are

$$\sigma_k^2 = \frac{\sum \bar{v}_j \bar{v}_j}{n-4} \qquad /75/$$

$$\alpha_k = \frac{\sum \bar{v}_j \bar{v}_{j+1}}{\sum \bar{v}_j \bar{v}_j} \qquad /76/$$

The residuals for which $v_j > q \cdot \sigma_k$ /q=2.5,3.0/ are deleted and the adjustment is repetead. If no further residuals are to be rejected the adjustment is repeated till convergence /for latitude: 0.02", longitude: 0.02" $\cos\varphi$, height: 0.7 m and frequency offset: 0.04 Hz/.

As a by-product, the station coordinates are averaged from the single pass solutions:

$$\varphi = \frac{\sum(\varphi^c + d\varphi_k)}{n_{pass}}$$

$$\lambda = \frac{\sum(\lambda^c + d\lambda_k)}{n_{pass}} \qquad /77/$$

$$H = \frac{\sum(H^c + dH_k)}{n_{pass}}$$

where n_{pass} is the number of all accepted passes, φ^c, λ^c, H^c are preliminary station coordinates, $d\varphi_k$, $d\lambda_k$, dH_k are the corrections computed from a single pass k and $k=1,2,\ldots,n_{pass}$.

6.2. SINGLE STATION SEQUENTIAL SOLUTION

In the SPPENC program the station coordinate determination is based on least squares solution for a single station from accumulated effect of observations of separate passes. It is a sequential solution. Here the corrections to station coordinates, to satellite initial position and to frequency offset are introduced as unknown parameters. The range difference semi-short arc observational equation for j-th observation of an i-th pass is as follows:

$$\bar{a}_{1j}d\varphi+\bar{a}_{2j}d\lambda+\bar{a}_{3j}dH+b_{1j}dX_i+b_{2j}dY_i+b_{3j}dZ_i+c_{1j}df_i = l_j+v_j \qquad /78/$$

Here $\bar{a}_{1j}$, $\bar{a}_{2j}$, $\bar{a}_{3j}$ are computed according to Eq. 39; b_{1j}, b_{2j}, b_{3j} are computed as prescribed for Eq. 55; c_{1j} and l_j are computed as given for Eq. 30. In every case epochs T'_{1j} and T'_{5j} are used instead of T_{1j} and T_{5j}.

Considering the system of observational equations of a pass, let us denote the station unknowns by vector X and the respective coefficients by matrix A as well as the orbital plus frequency unknowns by Y and the respective coefficients by B. Then the system of observational equations taken over k passes will have a form:

$$\begin{bmatrix} A_1 & B_1 & O & \cdots & O \\ A_2 & O & B_2 & \cdots & O \\ \vdots & \vdots & \vdots & & \vdots \\ A_k & O & O & \cdots & B_k \end{bmatrix} \begin{bmatrix} X \\ Y_1 \\ Y_2 \\ \vdots \\ Y_k \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_k \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \end{bmatrix} \qquad /79/$$

The system of normal equations obtained from Eq. 79 /supposing that the weight matrix **for observations is a unit one/ is as follows:**

$$\begin{bmatrix} \sum\dot{N}_i & \bar{N}_1 & \bar{N}_2 & \cdots & \bar{N}_k \\ \bar{N}_1^T & \ddot{N}_1 & O & \cdots & O \\ \bar{N}_2^T & O & \ddot{N}_2 & \cdots & O \\ \vdots & \vdots & \vdots & & \vdots \\ \bar{N}_k^T & O & O & & \ddot{N}_k \end{bmatrix} \begin{bmatrix} X \\ Y_1 \\ Y_2 \\ \vdots \\ Y_k \end{bmatrix} + \begin{bmatrix} \sum\dot{C}_k \\ \ddot{C}_1 \\ \ddot{C}_2 \\ \vdots \\ \ddot{C}_k \end{bmatrix} = O \qquad /80/$$

where $i= 1,2,\ldots,k$ and

$$\dot{N}_i = A_i^T A_i$$
$$\bar{N}_i = A_i^T B_i$$
$$\ddot{N}_i = B_i^T B_i + W_{orb}$$
$$\dot{C}_i = A_i^T L_i \qquad /81/$$
$$\ddot{C}_i = B_i^T L_i$$

with orbit constraint matrix

$$W_{orb} = \text{Transform}\left(W_k \left(26{,}105 \text{ m}\right)\right)$$

Eq. 80 is a first order partitioned system. For its reduction /Brown, Trotter 1969/ proposed a procedure. The following computation way is based on this procedure of reduction being performed in a pass-by-pass manner.

For an i-th pass:

1. The submatrices $\dot{N}_i, \bar{N}_i, \ddot{N}_i$ and the subvectors $\dot{C}_i, \ddot{C}_i$, are computed by Eq. 81.
2. A work matrix is produced: $Q_i = \bar{N}_i \ddot{N}^{-1}$ /82a/
3. The submatrix $\dot{N}_i$ and the effect coming from $\bar{N}_i$ and $\ddot{N}_i$ are added to the reduced normal matrix accumulated in the previous i-1 passes:

$$\dot{M}_i = \dot{M}_{i-1} + \left(\dot{N}_i - Q_i \bar{N}_i^T\right) \qquad /82/$$

where

$$\dot{M}_{i-1} = \left(\dot{N}_1 - Q_1 \bar{N}_1^T\right) + \left(\dot{N}_2 - Q_2 \bar{N}_2^T\right) + \ldots + \left(\dot{N}_{i-1} - Q_{i-1} \bar{N}_{i-1}^T\right)$$

The same operation is performed over constant vectors $\dot{C}_i$ and $\ddot{C}_i$:

$$C_i = C_{i-1} + \left(\dot{C}_i - Q_i \ddot{C}_i\right) \qquad /83/$$

where

$$C_{i-1} = \left(\dot{C}_1 - Q_1 \ddot{C}_1\right) + \left(\dot{C}_2 - Q_2 \ddot{C}_2\right) + \ldots + \left(\dot{C}_{i-1} - Q_{i-1} \ddot{C}_{i-1}\right)$$

Here $\dot{M}_i$ and C_i are the reduced normal equations and reduced constant vector.

4. The i-th estimation of the unknown vector composed of the station coordinate corrections is computed:

$$x_i = \dot{M}_i^{-1} c_i \qquad /84/$$

5. The sum of the residual squares and of the degree of freedom is computed for the phase of the given pass:

$$\left(v^T v\right)_i = \left(v^T v\right)_{i-1} + v_i^T v_i$$

$$f_i = f_i - 1 + \left(n_i - 1\right)$$

where for the previous i-1 passes /85/

$$\left(v^T v\right)_{i-1} = v_1^T v_1 + v_2^T v_2 + \dots + v_{i-1}^T v_{i-1}$$

$$f_{i-1} = 3 + \left(n_1 - 1\right) + \left(n_2 - 1\right) + \dots + \left(n_{i-1} - 1\right)$$

An estimation for the standard deviation of the estimated station coordinates obtained after i passes is as follows:

$$\begin{matrix} \sigma^2_{d\varphi,i} \\ \sigma^2_{d\lambda,i} \\ \sigma^2_{dH,i} \end{matrix} = \frac{\left(v^T v\right)_i}{f_i} \dot{M}_i^{-1} \qquad /86/$$

After introducing the observations of a new pass and proceeding with formulas in Eq. 81-86 a new estimation is obtained for the station coordinates and their standard deviation which is better then the previous ones. In statistical sence, the best estimation is obtained after the last pass to be processed to get single station coordinates in question.

6.3. MULTISTATION SHORT ARC SOLUTION

6.3.1. Reduction of second order system

In the SADOSA program different adjustment models can be used. Let us select one of them characterized by observational equations Eq. 62 written for every observations of each event ik /$i=1,2,\ldots,n_{st}$ are the stations participating in the network; $k=1,2,\ldots,m$ are the orbits observed in the network/.

The matrix of normal equations generated by design matrix of the total system will have the following form:

$$\begin{bmatrix} \hat{U} & \tilde{U}_1 & \tilde{U}_2 & \cdots & \tilde{U}_m \\ \tilde{U}_1^T & \ddot{U}_1 & 0 & \cdots & 0 \\ \tilde{U}_2^T & 0 & \ddot{U}_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{U}_m^T & 0 & 0 & \cdots & \ddot{U}_m \end{bmatrix} \begin{bmatrix} \hat{d} \\ d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix} = \begin{bmatrix} \hat{C} \\ C_1 \\ C_2 \\ \vdots \\ C_m \end{bmatrix} \qquad /87/$$

which seems to have the same structure as the first order system in Section 6.2. However Eq. 87 has a finer structure. The submatrices $\tilde{U}_k$, $\ddot{U}_k$ and the subvectors d_k, C_k are of the form

$$\tilde{U}_k = \left[\dot{U}_k \bar{U}_{k1} \bar{U}_{k2} \;\cdots\; \bar{U}_{kn_k}\right]$$

$$d_k = \left[\dot{d}_k \ddot{d}_{k1} \ddot{d}_{k2} \;\cdots\; \ddot{d}_{kn_k}\right]^T$$

$$C_k = \left[\dot{C}_k \ddot{C}_{k1} \ddot{C}_{k2} \;\cdots\; \ddot{C}_{kn_k}\right]^T \qquad /88/$$

$$\ddot{U}_k = \begin{bmatrix} \dot{N}_k & \bar{N}_{k1} & \bar{N}_{k2} & \cdots & \bar{N}_{kn_k} \\ \bar{N}_{k1}^T & \ddot{N}_{k1} & 0 & \cdots & 0 \\ \bar{N}_{k2}^T & 0 & \ddot{N}_{k2} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \bar{N}_{kn_k}^T & 0 & 0 & \cdots & \ddot{N}_{kn_k} \end{bmatrix}$$

that is they are also of first order partitioned systems. Thus, the matrix of normal equations [illegible] the total observational network is second order partitioned one.

The properties and reduction of such second order partitioned system are given by /Brown, Trotter 1969/. In the SADOSA program the reduction procedure suggested by /Brown, Trotter 1969/ has been adopted. It is given in the followings.

The reduction procedure is carried out in an outer and an inner loops. The outer loop handles the k-th orbit of the network. Inside outer loop of the k-th orbit an inner loop eliminates the error model unknowns of each consecutive station with index i /$i=1,2,\ldots,n_k$/ observing the given orbit, that is, performs a first order reduction. As the inner loop is finished, the outer loop eliminates the orbital unknowns of the k-th orbit, that is, performs a second order reduction. As the outer loop is finished, a reduced normal equation system is obtained with station unknowns only.

This reduction procedure is expressed by formulas of the following steps:

1. The design matrix is formed for the station i of the orbit k:

$$\hat{B}_{ik}\hat{d}_i+\dot{B}_{ik}\dot{d}_k+\ddot{B}_{ik}\ddot{d}_{ik} = L_{ik}+V_{ik} \qquad /89/$$

   where the observational vector L_{ik} is associated with weight matrix P_{ik} computed as shown in Section 5.2.

2. The normal equation matrix N_{ik} is generated and devided into the respective submatrices for the given event ik:

$$N_{ik} = \begin{bmatrix} \hat{U}_{ik} & \dot{U}_{ik} & \bar{U}_{ik} & \hat{C}_{ik} \\ \dot{U}^T_{ik} & \dot{N}_{ik} & \bar{N}_{ik} & \dot{C}_{ik} \\ \bar{U}^T_{ik} & \bar{N}^T_{ik} & \ddot{N}_{ik} & \ddot{C}_{ik} \end{bmatrix} = \begin{bmatrix} \hat{B}^T_{ik} \\ \dot{B}^T_{ik} \\ \ddot{B}^T_{ik} \end{bmatrix} P_{ik} \begin{bmatrix} \hat{B}_{ik}\dot{B}_{ik}\ddot{B}_{ik}\ddot{C}_{ik} \end{bmatrix} \qquad /90/$$

3. The error model constraints are considered and two work matrices are computed for the ik-th event:

$$\begin{aligned} W1_{ik} &= \bar{U}_{ik}\left(\ddot{N}_{ik}+W_{ik}\right)^{-1} \\ W2_{ik} &= \bar{N}_{ik}\left(\ddot{N}_{ik}+W_{ik}\right)^{-1} \end{aligned} \qquad /91/$$

   with diagonal matrix of error model constraints W_{ik} given by

Eq. 63.

4. The first order reduction is performed for the ik-th event:

$$\begin{bmatrix} [\dot{N}]_{ik} & [\bar{N}]_{ik} & [\dot{C}]_{ik} \\ [\bar{N}]^T_{ik} & [\ddot{N}]_{ik} & [\ddot{C}]_{ik} \end{bmatrix} = \begin{bmatrix} \hat{U}_{ik} & \dot{U}_{ik} & \hat{C}_{ik} \\ \dot{U}^T_{ik} & \dot{N}_{ik} & \dot{C}_{ik} \end{bmatrix} - \begin{bmatrix} W1_{ik} \\ W2_{ik} \end{bmatrix} \begin{bmatrix} \bar{U}^T_{ik} \bar{N}^T_{ik} \ddot{C}_{ik} \end{bmatrix} \quad /92/$$

5. Submatrices $(\ddot{N}_{ik}+W_{ik})^{-1}$, $\ddot{C}_{ik}$ and work matrices $W1_{ik}$, $W2_{ik}$ are stored for use in back solution.

6. As each reduced array of Eq. 92 is formed it is added to the sum of its predecessors there-by producing, after running through $i=1,2,\ldots,n_k$,

$$\begin{bmatrix} [N]_k & [\bar{N}]_k & [\dot{C}]_k \\ [\bar{N}]^T_k & [\ddot{N}]_k & [\ddot{C}]_k \end{bmatrix} = \begin{bmatrix} \sum[\dot{N}]_{ik} & \sum[\bar{N}]_{ik} & \sum[\dot{C}]_{ik} \\ \sum[\bar{N}]^T_{ik} & \sum[\ddot{N}]_{ik} & \sum[\ddot{C}]_{ik} \end{bmatrix} \quad /93/$$

7. As each array of the form Eq. 93 is produced /that is the inner loop is finished/, the second order reduction is performed for the k-th orbit:

$$\begin{bmatrix} [S]_k & [C]_k \end{bmatrix} = \begin{bmatrix} [\dot{N}]_k & [\dot{C}]_k \end{bmatrix} - W3_k \begin{bmatrix} [\bar{N}]^T_k & [\ddot{C}]_k \end{bmatrix} \quad /94/$$

with work matrix

$$W3_k = [\bar{N}]_k \left([\ddot{N}]_k + W_k\right)^{-1} \quad /95/$$

where W_k is the diagonal matrix of orbital unknown constraints given by Eq. 64a for broadcast ephemeris or Eq. 64b for precise ephemeris.

8. Submatrices $([\ddot{N}]_k+W_k)^{-1}$, $[\ddot{C}]_k$ and work matrix $W3_k$ are stored for use in back solution.

9. As each array of Eq. 94 is formed it is added to the sum of its predecessors thereby producing, after running through $k=1,2,\ldots,m$,

$$\begin{bmatrix} [S] & [C] \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{m} [S]_k & \sum_{k=1}^{m} [C]_k \end{bmatrix} \quad /96/$$

Its worths to note that the summation in Eq. 96 includes also a sorting procedure by station.

10. Constraints of station unknowns are considered using constraint matrix W given by Eq. 65 or/and Eq. 65a or inner constraint. As result, the reduced normal equation system is obtained with respect to station unknowns:

$$\begin{bmatrix} N & U \end{bmatrix} = \begin{bmatrix} [S] & [C] \end{bmatrix} + \begin{bmatrix} W & \phi \end{bmatrix} \qquad /97/$$

6.3.2. Back solution of second order partitioned system

First, the station unknowns are obtained. Then outer and inner loops will operate. In an outer loop the orbital unknowns are computed. In an inner loop for k-th orbit the error model unknowns are computed. As a byproduct the respective contributions to the constraint misclosure $XP_X X$ are computed.

The back solution is expressed by formulas of the following steps:

1. The station unknowns:

$$\hat{d} = - N^{-1} U \qquad /98/$$

and the constraint misclosure for station unknowns /VPVC1/:

$$\left(XP_X X\right)_{sta} = \hat{d}^T W \hat{d} \qquad /99/$$

2. Repeate 2.1 and 2.2 m times /outer loop/:

2.1. Read the stored $\left([\ddot{N}]_k + W_k\right)^{-1}, [\ddot{C}]_k$ and $W3_k$ of k-th orbit.

$$\dot{d}_k = \left([\ddot{N}]_k + W_k\right)^{-1} [\ddot{C}]_k - W3_k \hat{d} \qquad /100/$$

and the constraint misclosure weighted square after the k-th orbital unknowns to be accumulated /VPVC2/:

$$\left(XP_X X\right)_{orb,k} = \left(XP_X X\right)_{orb,k-1} + \dot{d}_k^T W_k \dot{d}_k \qquad /101/$$

2.2. Repeate 2.2.1. n_k times /inner loop/:

2.2.1. Read the stored $\left(\ddot{N}_{ik} + W_{ik}\right)^{-1}$, $\ddot{C}_{ik}$, $W1_{ik}$ and $W2_{ik}$.

The ik-th error model unknowns:

$$\ddot{d}_{ik} = \left(\ddot{N}_{ik} + W_{ik}\right)^{-1} \ddot{C}_{ik} - W2_{ik} \dot{d}_k - W1_{ik} \hat{d} \qquad /102/$$

and the constraint misclosure weighted square after the ik-th error model unknowns to be accumulated /VPVC3/:

$$(XP_X X)_{err,j} = (XP_X X)_{err,j-1} + \ddot{d}^T_{ik} W_{ik} \ddot{d}_{ik} \qquad /103/$$

where $j = \sum_{\ell=1}^{k-1} n_\ell + i$

6.3.3. Standard deviation of unit weight

Estimation of the residuals V_{ik} in Eq. 89 is obtained from the observational vectors computed in the next iteration /or, if last, in an iteration-like procedure/. Thus sum of squares of the weighted residuals of observations is obtained /VPVOB/:

$$VPV_{obs} = \sum_{k=1}^{m} \sum_{i=1}^{n_k} V^T_{ik} P_{ik} V_{ik} \qquad /104/$$

so that $V_{ik} = L^{iter+1}_{ik}$

The constraint misclosure weighted square summed up over all contraints:

$$XP_X X = (XP_X X)_{sta} + (XP_X X)_{orb} + (XP_X X)_{err} \qquad /105/$$

The total sum of squares of all weighted residuals /VPV/:

$$VPV = VPV_{obs} + XP_X X \qquad /106/$$

The degree of freedom coming from observations:

$$f_{obs} = \sum_{k=1}^{m} \left[\sum_{i=1}^{n_k} \left(n_{poz,ik} - n_{err} \right) - n_{orb} \right] - 3n_{sta} \qquad /107a/$$

The degree of freedom coming from constraints:

$$f_{constr} = c_{sta} + mn_{orb}c_{orb} + \sum_{k=1}^{m} \sum_{i=1}^{n_k} c_{ik} \qquad /107b/$$

The total degree of freedom:

$$f = f_{obs} + f_{constr}$$

or /107/

$$f = (c_{sta} - 3n_{sta}) + \sum_{k=1}^{m} \left[n_{orb}(c_{orb} - 1) + \sum_{i=1}^{n_k} (c_{ik} - n_{err} + n_{poz,ik}) \right]$$

where

c_{sta}	is the number of station positional constraints
n_{sta}	is the number of stations in the adjustment /NSTA/
m	is the number of orbits in the adjustment /NORB/
n_k	is the number of station observing a given orbit k
n_{orb}	is a factor /NXYZJK/, $n_{orb}=3$ if semi-short arc, $n_{orb}=6$ if short arc
c_{orb}	is a flag /IORBC/ specifying if the orbits are to be constrained $/c_{orb}=1/$, or not $/c_{orb}=0/$
c_{ik}	is the number of constraints in an event ik
n_{err}	is the number of error model parameters to be determined in the adjustment /NAIPR/ for an event ik
$n_{poz,ik}$	is the number of accepted observations in an event ik.

The internal standard deviation of unit weight:

$$\sigma_{oi} = \sqrt{\frac{VPV_{obs}}{f_{obs}}} \qquad /108/$$

The external standard deviation of unit weight:

$$\sigma_{oe} = \sqrt{\frac{XP_X X}{f_{constr}}} \qquad /109/$$

The a'posteriori standard deviation of unit weight of the network:

$$\sigma_o = \sqrt{\frac{VPV}{f}} \qquad /110/$$

6.3.4. Inversion of the reduced normals

The reduced normal equation matrix N /and the U vector/ /Eq. 97/ is handled so that only the upper triangle is stored in 3*3 size block format /and 3*1 for U/. The normal equations are solved by a Gauss reduction, the back solution and computation of the inverse is performed by method associated with the name of Banachiewicz.

The procedure given by /Mueller et al. 1973/ and the subroutine realization published in /Reilly et al. 1972/ is adopted for the SADOSA program.

6.3.5. Computation of station coordinates

An estimation of the station coordinate corrections is obtained:

$$\hat{d} = \left[dX_1, dY_1, dZ_1, \ldots, dX_{n_{sta}}, dY_{n_{sta}}, dZ_{n_{sta}},\right]^T \qquad /111/$$

The estimated rectangular station coordinates are computed as sum of the preliminary coordinates and the corrections:

$$\left[X_1, Y_1, Z_1, \ldots, X_{n_{sta}}, Y_{n_{sta}}, Z_{n_{sta}}\right]^T =$$

$$\left[X_1^c, Y_1^c, Z_1^c, \ldots, X_{n_{sta}}^c, Y_{n_{sta}}^c, Z_{n_{sta}}^c\right]^T + \hat{d} \qquad /112/$$

The estimated variance-covariance matrix of the rectangular station coordinates:

$$\Sigma_{\substack{X\\Y\\Z}} = \sigma_o^2 N^{-1} \qquad /113/$$

and the standard deviation of each rectangular station coordinate is obtained as square root from the respective diagonal elements of $\Sigma_{\substack{X\\Y\\Z}}$.

The corrections to ellipsoidal coordinates $d\varphi_i$ (in arcsec), $d\lambda_i$ (in arcsec), dH_i (in m) of a station i are computed from the respective i-th 3×1 block of $\hat{d}$ and by virtue of a G_i transformation matrix

$$\left[d\varphi_i,\ d\lambda_i,\ dH_i\right]^T = G_i \cdot \left[dX_i,\ dY_i,\ dZ_i\right]^T \qquad /114a/$$

where

$$G_i = \begin{bmatrix} -\frac{\rho}{R_i}\sin\varphi_i\cos\lambda_i & -\frac{\rho}{R_i}\sin\varphi_i\sin\lambda_i & \frac{\rho}{R_i}\cos\varphi_i \\ -\frac{\rho}{R_i}\frac{\sin\lambda_i}{\cos\varphi_i} & \frac{\rho}{R_i}\frac{\cos\lambda_i}{\cos\varphi_i} & 0 \\ \cos\varphi_i\cos\lambda_i & \cos\varphi_i\sin\lambda_i & \sin\varphi_i \end{bmatrix}$$

with ρ = 206 264.806" and R_i distance of the station from centre of the ellipsoid.

The ellipsoidal coordinates for station i are obtained as follows

$$\left[\varphi_i,\ \lambda_i,\ H_i\right]^T = \left[\varphi_i^c,\ \lambda_i^c\ \ H_i^c\right]^T + \left[d\varphi_i,\ d\lambda_i,\ dH_i\right]^T \qquad /114b/$$

The respective estimation of variance-covariance matrix of a station /units: m^2, $arcsec^2$, m arcsec/:

$$\Sigma_{\substack{\varphi i\\ \lambda i\\ Hi}} = G_i \cdot \Sigma_{\substack{Xi\\ Yi\\ Zi}} \cdot G_i^T \qquad /115/$$

The error ellipsoid parameters are also computed using a standard IBM procedure /subroutine DEIGEN/ to solve the task of eigenvectors. Based on the diagonal 3*3 blocks of the variance-covariance matrix, for every station i the eigenvalues λ_{jj} /j=1,2,3/ are obtained from

$$\left|\Sigma_{ii} - \lambda_{jj} I\right| = \phi \qquad /116/$$

By means of λ_{jj} the eigenvectors T^j /3 components/ are obtained from solution of

$$\left[\Sigma_{ii} - \lambda_{jj} I\right] T^j = \phi \qquad /117/$$

From each T^j the length, latitude and longitude of an error ellipsoid axis is computed. The latitudes and longitudes of the three axes are then transformed into the elevation above horizon and the azimuth.

6.3.6. Relative station positions

For each station-to-station combination the relative rectangular coordinates, the distance and the respective standard deviations as well as the variance-covariance matrix and error ellipsoid of relative positions are computed.
The relative rectangular coordinates:

$$\begin{bmatrix} DX_{ij} \\ DY_{ij} \\ DZ_{ij} \end{bmatrix} = \begin{bmatrix} X_j & - & X_i \\ Y_j & - & Y_i \\ Z_j & - & Z_i \end{bmatrix} \qquad /118/$$

with $i=1,2,\ldots,n_{sta}$, $j=i+1,i+2,\ldots,n_{sta}$

The distance

$$D = \left[(X_j-X_i)^2+(Y_j-Y_i)^2+(Z_j-Z_i)^2\right]^{1/2} \qquad /119/$$

The variance-covariance matrix of relative position:

$$\Sigma_{\substack{DXij\\DYij\\DZij}} = \Sigma_i + \Sigma_j - 2\,\Sigma_{ij} \qquad /120/$$

where Σ_i and Σ_j are the respective variance 3*3 diagonal bloks for station i and j as well as Σ_{ij} is the 3*3 block at crossing of i-th row and j-th column of 3*3 blocks /covariance block/.

The standard deviations of relative coordinates are computed as square root from the diagonal elements of $\Sigma_{\substack{DXij\\DYij\\DZij}}$.

The error ellipsoid parameters are computed in the same way as described in the previous section 6.35. Computation here is based on the 3*3 size $\Sigma_{\substack{DXij\\DYij\\DZij}}$ variance-covariance matrix.

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